Published by Institute of Physics Publishing for SISSA
Received: *August 22, 2005*
Accepted: *September 10, 2005*
Published: *September 20, 2005*

MSLED, neutrino oscillations and the cosmological constant

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Abstract: We explore the implications for neutrino masses and mixings within the minimal version of the supersymmetric large-extra-dimensions scenario (MSLED). This model was proposed in [10] to extract the phenomenological implications of the promising recent attempt (in [1]b) to address the cosmological constant problem. Remarkably, we find that the simplest couplings between brane and bulk fermions within this approach can lead to a phenomenologically-viable pattern of neutrino masses and mixings that is *also* consistent with the supernova bounds which are usually the bane of extra-dimensional neutrino models. Under certain circumstances the MSLED scenario can lead to a lepton mixing (PMNS) matrix close to the so-called bi-maximal or the tri-bimaximal forms (which are known to provide a good description of the neutrino oscillation data). We discuss the implications of MSLED models for neutrino phenomenology.

Keywords: Cosmology of Theories beyond the SM, Large Extra Dimensions, Neutrino Physics.

Contents

1. Introduction

The supersymmetric Large Extra Dimensional (SLED) scenario was proposed [1, 2] in an effort to provide the long-sought understanding of why the vacuum energy should have incredibly small value — $\rho_{\rm vac} = \lambda^4$ with $\lambda \sim 10^{-3}\,$eV — which is observed [3]. According to this proposal all observed particles reside on a 3-brane situated within an extra-dimensional world which has two large ($1/r \sim 10^{-2}\,$eV) dimensions[1] (plus possibly others which are much smaller, $1/r \gtrsim 1\,$TeV), which can be possible [4] provided the scale of gravity in the large extra dimensions is of order the TeV scale.

The rationale for such a picture is that although non-gravitational (brane) particles and interactions behave in the usual way, the gravitational response to their zero-point vacuum energy changes because gravity becomes 6-dimensional at energies above $10^{-2}\,$eV. Although the ultimate success of the proposal is still under active study [5 − 7], its prospects for success remain promising.

[1] In our conventions the extra-dimensional volume is $V_2 = r^2$, and so (for a square 2-torus, for example) the Kaluza-Klein (KK) masses are of order $m_{KK} \sim 2\pi/r$.

A particularly attractive feature of the SLED scenario is that it is very predictive, with many testable implications for cosmology and for tests of gravity on large and small scales. Better yet, it has observable implications for high-energy collider experiments, whose detailed implications have only recently begun to be explored [8, 9]. The predictiveness of the SLED picture is most pronounced in its minimal version, MSLED [10], which assumes the particle content on our brane to be given by precisely the Standard Model particle content.

Because of the similarity between the Kaluza-Klein (KK) scale in these models, $2\pi/r \sim 10^{-1}$ eV, and the neutrino masses which are observed in oscillation experiments, it is natural to examine whether these models might also have observational implications for neutrino physics. We examine this issue here, and find a rich set of phenomena can arise within the neutrino sector. In particular, we argue that the observed pattern of neutrino masses and mixings can emerge through the mixing of the brane-bound neutrinos with the many massless 6D fermions which SLED models predict must appear in the bulk, without running afoul of the bounds on sterile neutrinos.

1.1 Neutrino masses in MSLED

The possibility that large extra dimensions might have implications for neutrino masses has been explored with many variations [11 − 15]. The vast majority of these variations share the common feature that the neutrino mass eigenstates arise when the usual brane-bound neutrino flavours mix with various higher-dimensional neutral fermions which are assumed to reside within the bulk. It is generically assumed that these bulk fermions are massless in the higher-dimensional sense, and so from the 4-dimensional point of view only acquire masses (before mixing with the brane neutrinos) through the Kaluza-Klein mechanism. Various models are obtained by adjusting the strength of the brane-bulk couplings, the number of extra dimensions and the size of each dimension.

We follow a similar construction here, but find that the results from supersymmetric large extra dimensions are much more predictive, for the following reasons.

- The massless field content in the bulk is dictated by 6-dimensional supersymmetry, and automatically contains numerous candidate neutral fermion fields which can mix with the brane neutrinos. Better yet, there is an understanding why the 6D mass of these bulk fermions must vanish. They vanish because they are related by supersymmetry to massless bosons, such as the graviton.

- There is little freedom to alter the size and shape of the large extra dimensions, because the value of the observed Dark energy density dictates that there are two large dimensions, and the spacing of KK masses in *both* of these dimensions is not larger $m_{KK} \sim 2\pi/r \sim 0.1\,\mathrm{eV}$.

- Like for other models with large extra dimensions, there is a natural extension of lepton number to extra-dimensional fermions — chirality in the two large dimensions — which is also naturally broken at the scale m_{KK}. This allows a technically natural understanding of why quantum corrections do not destabilize the observed neutrino mass pattern.

- The back-reaction of the branes onto the bulk geometry is important in SLED models because it is part of the mechanism for suppressing the vacuum energy. In particular, this is how the bulk learns about supersymmetry breaking on the branes because the boundary conditions which the branes set up for the bulk fermions typically remove the massless fermionic Kaluza-Klein modes (like the gravitini) from the spectrum. This absence of bulk-fermion zero modes also removes the massless modes from the bulk fermions which mix with the brane neutrinos. This is important phenomenologically, since it is the degeneracy (in the absence of brane-bulk couplings) between these massless bulk states with the massless brane states which typically leads to large mixings between these states once brane-bulk couplings are turned on. In this way SLED models sidestep one of the phenomenological difficulties of large-extra-dimensional neutrino models.[2]

Remarkably, we find that this restrictiveness can nonetheless very naturally lead to a phenomenologically successful picture of neutrino mixing.

1.2 Required neutrino properties

Before describing the neutrino mass and mixing pattern to which SLED models lead, we first pause to summarize the main results of current neutrino phenomenology which must be explained.

Active neutrino oscillations. The observed pattern of neutrino oscillations are consistent with there being just the 3 active neutrino species, whose participation in the charged-current weak interactions has the form

$$\mathcal{L}_{cc} = \frac{ig\,U_{ai}}{\sqrt{2}}\, W_\mu \, (\overline{\ell}_a \gamma^\mu \, \gamma_L \, \nu_i) + \text{h.c.}\,, \tag{1.1}$$

where γ_L is the projector onto left-handed spinors and $i = 1, 2, 3$ labels the 3 neutrino types, as $a = 1, 2, 3$ does for the leptons: $\{\ell_1, \ell_2, \ell_3\} = \{e, \mu, \tau\}$. Here the *PMNS* matrix [16], U_{ai}, describes the amplitude with which the neutrino type 'i' reacts with the charged lepton 'a'. For three types of neutrinos it may be parameterized in terms of mixing angles and phases: $U = V\,K$, with

$$\begin{aligned} V &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned} \tag{1.2}$$

and

$$K = \begin{pmatrix} e^{i\rho} & & \\ & e^{i\sigma} & \\ & & 1 \end{pmatrix}, \tag{1.3}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

[2] Back-reaction issues are included in the more detailed microscopic construction of ref. [13].

As is well known, assuming only three neutrinos to be relevant implies the splitting between two of these neutrinos must be much smaller than their common splitting from the third, and we follow common practice by choosing our labelling for the neutrino eigenstates such that the small splitting is between ν_1 and ν_2. With this choice, the successful description of the oscillation lengths seen in solar- and atmospheric-neutrino experiments implies the three neutrino masses, $m_{1,2,3}$, must satisfy [17, 18]

$$\begin{aligned} \Delta m^2_{atm} &= |m_3^2 - m_2^2| = (1.1 - 3.4) \times 10^{-3}\ \mathrm{eV}^2\,, \\ \Delta m^2_{\odot} &= |m_2^2 - m_1^2| = (5.4 - 9.4) \times 10^{-5}\ \mathrm{eV}^2\,. \end{aligned} \tag{1.4}$$

This range for solar neutrinos corresponds to the Large-Mixing-Angle (LMA) solution, which is the only one which accounts for the most recent results from the SNO and KamLAND oscillation measurements. In themselves neutrino oscillations do not fix the values of each of the masses separately, leaving open two possibilities. If the degenerate pair, ν_1 and ν_2, are less massive than ν_3, then the mass pattern is known as a 'normal' hierarchy, while if $m_1, m_2 > m_3$ the hierarchy is 'inverted'.

With this choice, the absence of the observation of short-distance oscillations of reactor-generated neutrinos implies the 3σ limit

$$|U_{e3}|^2 = s_{13}^2 < 0.067\,. \tag{1.5}$$

Because this implies $c_{13} > 0.966$, we have $1 > |U_{e1}|^2 + |U_{e2}|^2 = c_{13}^2 > 0.933$ and so it is convenient to write $|U_{e1}| = c_{12}c_{13} \simeq \cos\theta_\odot$ and $|U_{e2}| = s_{12}c_{13} \simeq \sin\theta_\odot$ when describing other oscillation measurements. Observations of solar-neutrino oscillations constrain s_{12}, and imply

$$0.67 \le \sin^2 2\theta_\odot \le 0.93\,, \tag{1.6}$$

where, as above, $\theta_\odot \simeq \theta_{12}$. Oscillations as seen in atmospheric neutrinos are close to maximal, and satisfy

$$0.85 \le \sin^2 2\theta_{\mathrm{atm}} \le 1\,, \tag{1.7}$$

where $\theta_{\mathrm{atm}} = \theta_{23}$.

Suggestive textures. For later convenience it is worth remarking that a good first description for the observed values of the elements of the PMNS matrix is obtained by taking $\theta_{13} \approx 0$ and so $s_{13} \approx 0$, $c_{13} \approx 1$. We may also approximate atmospheric-neutrino mixing as being maximal, in which case $\theta_{23} \approx \pi/4$ and $s_{23} \approx c_{23} \approx 1/\sqrt{2} = 0.707\ldots$, so $\sin^2 2\theta_{\mathrm{atm}} = \sin^2 2\theta_{23} = 1$. With these choices the mixing matrix takes the form

$$U \approx \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{1.8}$$

These choices are easy to obtain within specific models like the extra-dimensional ones to be discussed shortly, because they follow as consequences of the assumption that the masses are invariant under a discrete Z_2 symmetry [19–21] which interchanges the second and third generations. To see this, notice that the most general 3×3 mass matrix which is invariant under this interchange may be written

$$M_{Z_2} = \begin{pmatrix} m & m' & m' \\ m' & m+\hat{m} & \hat{m} \\ m' & \hat{m} & m+\hat{m} \end{pmatrix} , \tag{1.9}$$

and this matrix always admits the eigenvector $\nu \propto (0,1,-1)^T$ with eigenvalue m. As we shall see, imposing this symmetry on extra-dimensional models gives a matrix of this form, but with $m = 0$, leading to an inverted-hierarchy mass pattern for which ν_3 is massless.

In later sections we shall be led towards two particular choices of mixing matrices whose entries are very similar (up to signs) to the form of eq. (1.8). For one case $s_{12} \approx -1/\sqrt{3} = -0.577\ldots$ and $c_{12} \approx \sqrt{2/3} = 0.816\ldots$ (and so $\sin^2 2\theta_{12} = 4s_{12}^2 c_{12}^2 = 8/9 = 0.888\ldots$), which is called the tri-bimaximal-mixing form [22], and lies within the observed range allowed by oscillation measurements. In this case the PMNS matrix becomes

$$U \approx \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} . \tag{1.10}$$

A second example to which later sections lead is the bi-maximal mixing form, for which $-s_{12} = c_{12} = 1/\sqrt{2}$, and so for which the PMNS matrix becomes

$$U \approx \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & 1/\sqrt{2} \\ -1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} . \tag{1.11}$$

This form does *not* successfully describe the data, because the maximal value $\sin^2 2\theta_\odot = 4s_{12}^2 c_{12}^2 = 1$ lies outside of the experimentally-allowed range. However it naturally arises if ν_1 and ν_2 are 'pseudo-Dirac', or carry an approximately-conserved lepton number [23]. In this case it is the perturbations which break the relevant lepton number which change s_{12} and bring the resulting PMNS matrix into agreement with observations.

Bounds on individual sterile neutrinos. Any 4D fermion which transforms as a singlet under the Standard Model gauge group can become a sterile neutrino if it mixes with any of the three active neutrinos. If such mixings exist then the PMNS matrix, U_{ai}, for the charged-current interactions is no longer square, since it has 3 rows ($a = 1,2,3$, for each charged lepton) but $3+N$ columns ($i = 1,\ldots,3+N$ if the 3 active neutrinos are supplemented by N sterile counterparts). By virtue of these new mixing matrix elements sterile neutrinos can have observable signatures, none of which have been observed to date. This section summarizes the observational bounds which follow from these potential signatures.

Many of the phenomenological constraints on sterile neutrinos are performed under the minimal assumption that only a single species of sterile neutrino exists (i.e. $N = 1$) [24–26].

The bounds which result typically rely on the absence of oscillations between this neutrino and the usual active ones, and so the strength of the allowed mixing can depend sensitively on what is assumed about the mass difference, Δm^2_{14}, between the sterile and the relevant active neutrino state and about the active-sterile mixing parameter, $\varepsilon \sim |\sin\theta_s| \ll 1$.[3] We summarize some of these bounds here, and return in later sections to describe the somewhat stronger bounds which arise for the higher-dimensional models of present interest.

- *Solar neutrinos:* the agreement between solar-model predictions and the observed charged-current and neutral-current solar-neutrino fluxes at SNO constrain the amount of flux which can be lost to sterile states. For generic neutrino mass differences larger than $10^{-12}\,\mathrm{eV}^2$ these bounds constrain the sterile-active neutrino mixing angle to be smaller than $\varepsilon \lesssim 0.1$, where the precise bound depends on the mass difference. This bound improves to $\varepsilon \lesssim 0.001$ for mass splittings in a narrow range around $\Delta m^2_{14} = 10^{-4}\,\mathrm{eV}^2$, and to $\varepsilon \lesssim 0.01$ for $10^{-8}\mathrm{eV}^2 \lesssim \Delta m^2_{14} \lesssim 10^{-6}\,\mathrm{eV}^2$, due to the presence in these cases of resonant active-sterile MSW oscillations within the Sun.

- *Reactor neutrinos:* the absence of neutrino flux disappearance from reactor-generated neutrinos constrains $\varepsilon \lesssim 0.1$ for $\Delta m^2_{14} \gtrsim 10^{-3}\,\mathrm{eV}^2$.

- *Atmospheric neutrinos:* active-sterile neutrino oscillations can be excluded for $\varepsilon \lesssim 0.2$ for the range $10^{-4} \lesssim \Delta m^2_{14} \lesssim 10^{-2}\,\mathrm{eV}^2$.

- *Supernova neutrinos:* excessively large active-sterile anti-neutrino conversions within supernovae can drain flux from the supernova neutrino signal, and too much of this can be excluded from the observed signal from SN1987a. For resonant $\nu_e - \nu_s$ oscillations, this can constrain mixings down to $\varepsilon \lesssim 0.01$ for mass splittings, Δm^2_{14}, which are larger than around $10\,\mathrm{eV}^2$. However because neutrino densities themselves play a significant role in these resonant oscillations, there is a feedback mechanism through which efficient resonant active-sterile oscillations can change the neutrino densities and thereby rapidly turn off the resonance [26, 28].

- *Nucleosynthesis:* the dominant cosmological constraint for sterile neutrinos whose masses are smaller than $1\,\mathrm{eV}$ comes from the requirement that there not be too many light degrees of freedom contributing to the universal expansion during Big-Bang Nucleosynthesis (BBN). Even if the primordial abundance of sterile neutrinos is small, this constrains the strength of active-sterile oscillations because these can cause too efficient production of sterile neutrinos from the equilibrated active neutrinos. This latter condition can constrain neutrino abundances down to mass differences of order $10^{-8}\,\mathrm{eV}^2$, with the strongest constraints (coming for the largest mass differences) being of order $\varepsilon \lesssim 0.1$.

[3]The presence of sterile neutrinos can also modify non-neutrino precision electroweak measurements, such as by modifying the muon lifetime from which the Fermi constant is inferred [27], although these are not yet competitive with more direct observables.

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- *Cosmic microwave background:* measurements of CMB temperature fluctuations constrain the energy density which can be present in neutrinos at the epoch of recombination, leading to the constraint $\Omega_\nu h^2 < 10^{-2}$. Since this constrains the energy density present in neutrinos, the mixing angles to which this allows constraints are larger for heavier neutrinos, being sensitive to mixing angles as small as $\varepsilon \sim 0.001$ for sterile neutrino masses of order 10 eV.

Bounds on extra-dimensional sterile neutrinos. Since large-extra-dimensional models contain entire KK towers of sterile neutrinos, much more restrictive bounds are possible. For the purposes of discussing these bounds we anticipate the next section's results, and regard the sterile states to have masses $m_\ell \approx c_\ell/r$, and mixings ε_ℓ with active neutrino flavours, where ℓ labels the modes and c_ℓ and ε_ℓ are dimensionless parameters which depend on the details of the compactification. As we shall see from explicit diagonalizations, typically $\varepsilon_\ell \sim g/c_\ell$ where g is a constant which is independent of ℓ.

The bounds in this case come in two forms. First, the presence of enormous numbers of sterile KK modes having masses ranging upwards from $m_{KK} \sim 2\pi/r$, opens the possibility of there being active-sterile neutrino oscillations into specific sterile states for a great variety of oscillation lengths. This allows the use of the best of the bounds listed above for individual sterile neutrinos, provided only that the relevant mass differences are included amongst the allowed KK neutrino masses.

The second kind of bound relies on the enormous available phase space which the extra dimensions make available, since this allows bounds to be obtained from the absence of processes which radiate energy into sterile neutrinos *incoherently*, rather than through coherent active-sterile neutrino oscillations. These incoherent neutrino bounds are in addition to the similar bounds which constrain the amount of incoherent emission which may be tolerated into other species of bulk particles, such as gravitons. A closely related class of constraints come from the cosmological limits on the total energy which can be tied up at present in sterile neutrinos. These bounds are particularly potent for extra-dimensional models, for which enormous numbers of massive and stable sterile neutrino states can be present.

Bounds of these second type are normally used to completely exclude the possibility of explaining neutrino masses in terms of more than a single large extra dimension, since each additional large dimension enormously increases the amount of phase space which is available in bulk neutrino states [29 – 34]. The main novelty of our results in this paper is our ability to provide a phenomenologically viable description of neutrino masses using *two* large extra dimensions without running afoul of these energy-loss bounds.

Amongst the most restrictive new bounds obtained in this way are

- *Supernova bounds:* the large number of KK states makes energy-loss bounds from supernovae very strong. Each KK mode is radiated incoherently (i.e. by vacuum oscillations) with rate $\Gamma_i \sim (m/E)^2\Gamma_\nu$, where $m \sim g/r$ is a measure of the mass term which mixes the active and sterile neutrinos (see the next section for precise definitions), E is the neutrino energy and Γ_ν is a typical active-neutrino emission rate. Since the number of sterile neutrino states having mass smaller than E is of order $(Er/2\pi)^n$ for n extra dimensions, an estimate for the total incoherent emission rate into sterile states for $n = 2$ is

$$\Gamma_{\rm inv} = \sum_i \Gamma_i \sim \left(\frac{g}{rE}\right)^2 \Gamma_\nu \left(\frac{Er}{2\pi}\right)^2 \sim \left(\frac{g}{2\pi}\right)^2 \Gamma_\nu \,. \tag{1.12}$$

Refs. [30, 32] require this to be at most 10^{-8} of the active-neutrino rate, Γ_ν, which leads to the conservative bound on the dimensionless bulk-brane mixing of order $g/2\pi \lesssim 10^{-4}$.

We call this a conservative bound because it relies fairly strongly on the present-day understanding of supernova explosions, which are arguably relatively poorly understood given the inability of current computer models to successfully explode a star.[4] Another approach is only to require that the rate of sterile-neutrino radiation not be more than of the same order as the active-neutrino signal, in which case only a comparatively weak constraint on g is obtained.

For supernovae, another potentially very dangerous mechanism for energy loss into sterile neutrinos is possible because the large number of neutrino states allows a succession of potential resonant oscillations which can depress the active-neutrino survival rate as it passes through the supernova environment. For small mixing each resonance is narrow and the survival probability can be understood as the product of that for each KK mode. Even though each KK mode mixes with an amplitude which is suppressed by the mode's mass, the resonance for each has essentially the same adiabaticity parameter, leading to a prohibitively large conversion rate even for extremely small brane-bulk mixing parameter g. Happily, as pointed out in refs. [26, 28], this constraint is too naive since the feedback of such oscillations onto the supernova neutrino densities is likely to quickly reduce the rate of energy loss in this channel to the vacuum-oscillation rate described above.

- *Late-epoch cosmology:* large-extra-dimensional models face well-known dangers with cosmology since the various KK states for the many bulk fields can cause problems with Big Bang Nucleosynthesis or with over-closure of the universe if they are too abundant [33]. These bounds can be avoided if the KK modes do not appreciably decay into photons and if they and their decay products are not too abundant at the BBN epoch. Although it remains a challenge to obtain a pre-BBN cosmology which gives sufficiently few relic KK modes in models with large extra dimensions, we put this issue aside here since its proper understanding must also await a hitherto missing study of the time-dependence of 6D cosmological background solutions. We here therefore follow the usual practice and assume there to be an acceptable number of KK modes at BBN, and focus specifically on the neutrino-related constraints which arise due to the possibility of populating KK sterile neutrinos through active-sterile oscillations from the known equilibrium abundance of active neutrinos.

[4] We thank Marco Cirelli and George Fuller for helpful conversations on this point.

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Constraints on the present-day energy density in neutrinos preclude there being a significant number of stable massive sterile neutrino states having masses greater than of order 1 eV and having an appreciable mixing with active neutrinos. Mixings are dangerous in this context because they can keep sterile neutrinos in equilibrium down to low temperatures, leading to too large late-time abundances. An efficient way to evade these bounds is to require all of the sterile neutrinos to thermally decouple at a temperature, T_D, above the QCD phase transition so their abundance can be diluted by the entropy release which occurs during this transition. This requires $T_D \gtrsim 1\,\text{GeV}$, where for each sterile neutrino species T_D is related to the active-sterile mixing parameter ε by the condition

$$\varepsilon^2 G_F^2 T_D^5 \sim \frac{T_D^2}{M_p}\,, \tag{1.13}$$

where $G_F \sim 10^{-5}\,\text{GeV}^{-2}$ is the Fermi coupling and $M_p \sim 10^{18}\,\text{GeV}$ is the Planck mass. The condition $T_D \sim 1/(\varepsilon^2 G_F^2 M_p)^{1/3} \gtrsim 1\,\text{GeV}$ leads to the constraint $\varepsilon \lesssim 10^{-4}$. In later sections we find $\varepsilon_\ell \sim g/c_\ell$, where g measures the dimensionless brane-bulk coupling and $c_\ell/r \gtrsim 2\pi/r$ is the mass of the KK mode in question. We see that the constraint is strongest for the lightest modes, and for modes with $c_\ell \sim 2\pi$ this constraint implies $g \lesssim 2\pi \times 10^{-4}$.

We see that both of the above constraints may be satisfied provided that the brane-bulk couplings satisfy $g \lesssim 2\pi \times 10^{-4}$, and for this reason we have the regime of small g in mind when we describe the neutrino mixings in more detail in the next section.

2. Neutrinos in MSLED

In MSLED neutrino masses can arise through mixing between the usual three brane-bound neutrinos, ν_a, and fermions in the bulk. In order to describe this mixing we first digress to summarize our conventions for 6D spinors.

2.1 6D spinors

In 6 dimensions the Dirac matrices are 8×8 matrices, which we take to have the following form: $\Gamma_M = \{\Gamma_\mu, \Gamma_m\}$, where $\mu = 0, 1, 2, 3$ and $m = 4, 5$ and

$$\Gamma_\mu = \gamma_\mu \otimes I_2 \qquad \text{and} \qquad \Gamma_m = \gamma_5 \otimes \tau_m\,. \tag{2.1}$$

Here γ_μ denotes the usual 4×4 Dirac matrices, while

$$\tau_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \text{and} \qquad \tau_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,. \tag{2.2}$$

In addition we define the usual 4- and 2-dimensional chirality projectors $\gamma_L = \frac{1}{2}(1 + \gamma_5)$, $\gamma_R = \frac{1}{2}(1 - \gamma_5)$ and $\tau_\pm = \frac{1}{2}(1 \pm \tau_3)$, with

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{2.3}$$

In terms of these matrices, 6D chiral spinors are distinguished by the eigenvalues of the matrix $\Gamma_7 = \gamma_5 \otimes \tau_3$.

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With these choices 6D spinors carry 4D and 2D indices, (u, α), where $u = 1, \ldots, 4$ and $\alpha = 1, 2$. It is useful to take the basis for these spinors to have definite chirality, since a 6D spinor with positive chirality, $\Gamma_7 \psi = +\psi$, has 4 complex components and decomposes into two 4D states for which the eigenvalues of the pair (γ_5, τ_3) are $(\lambda, s) = (+1, +1)$ and $(\lambda, s) = (-1, -1)$.

Once compactified, we see that each 6D fermion reduces to a Kaluza-Klein tower of 4D fermions. For example a collection of 6D $\Gamma_7 = +1$ Weyl fermions has a 4D expansion of the form

$$N^I(x, y) = \frac{1}{r} \sum_{s=\pm} \sum_{\ell} u_{\ell s}(y)\, n^I_{\ell s}(x)\,, \tag{2.4}$$

where ℓ labels the KK modes, with $u_{\ell s}(y)$ being 2-component spinors which are chosen to satisfy $\tau_3\, u_{\ell s} = s\, u_{\ell s}$. Consequently (because $\Gamma_7 N^I = +N^I$) the 4D spinors $n^I_{\ell s}$ have 4D chirality $\gamma_5\, n^I_{\ell s} = s\, n^I_{\ell s}$ (notice that in the above all spinor indices are suppressed). An overall factor of the size of the extra dimensions, $1/r$, has been extracted from the extra-dimensional fermion wave-functions, $u_{\ell s}(y)$, so that these can satisfy r-independent orthonormality relations.

2.2 Bulk-brane couplings

The lowest-dimension bulk-brane interaction between left-handed lepton fields on the branes and a collection of $\Gamma_7 = +1$ 6D fermions, N^I, has the form

$$S_{\text{int},N} = \int d^4x\, \Big[\lambda_{aI\alpha}(L^u_a N^I_{u\alpha})H + \text{c.c.}\Big]\,. \tag{2.5}$$

Here the $\lambda_{aI\alpha}$ are coupling constants having dimensions of inverse mass, which we expect to be roughly of order M_g^{-1} in size, where $M_g \sim 10\,\text{TeV}$ [10] is the gravity scale in the bulk. In this expression the $SU_L(2)$ gauge index is suppressed, while the index $a = 1, 2, 3$ labels fermion generations. Similarly $u = 1, \ldots, 4$ and $\alpha = 1, 2$ are the 4D and 2D spinor indices, which we here temporarily re-instate.

It is convenient to write the couplings in terms of definite 2D chirality, with

$$\lambda_{aI\alpha} = \sum_{s=\pm} \lambda^{(s)}_{aI}\, u^{(s)\dagger}_{\alpha}\,, \tag{2.6}$$

with $\tau_3\, u^{(s)} = s\, u^{(s)}$. If both $\lambda^{(+)}_{aI}$ and $\lambda^{(-)}_{aI}$ are nonzero for some choice for I and a then these brane-bulk couplings violate the 2D local Lorentz transformations generated by

$$J = \frac{i}{4}\,[\Gamma^4, \Gamma^5] = \frac{i}{4}\Big(I \otimes [\tau_4, \tau_5]\Big) = \frac{1}{2}\Big(I \otimes \tau_3\Big)\,, \tag{2.7}$$

which act on the 2D spinor index α. On the other hand, if all of the $\lambda^{(+)}_{aI}$ (or $\lambda^{(-)}_{aI}$) should vanish, then the coupling preserves this $O(2)$ symmetry provided that the brane fermions are also taken to be charged under it. Any such $O(2)$ symmetry would be naturally interpreted as a lepton-number symmetry for the neutrino mass matrix, since this is the only possible way such a symmetry can be extended to an invariance of the rest of the Standard Model brane couplings given the assumption of minimal particle content.

Models with bulk-brane neutrino mixing have been studied in detail in ref. [11 − 14], but this earlier round of model building differs from the present picture in several important ways.

Brane back-reaction: first, these earlier analyses ignore the brane back-reaction onto the bulk. However, we know quite generally in MSLED that the back-reaction of the brane (or branes) onto the bulk geometry induces boundary conditions for the bulk fermions at the brane position, and these remove the otherwise-massless Kaluza-Klein level [1]. (Indeed this is how the bulk modes in MSLED 'see' that supersymmetry is broken by the branes.)

Approximate lepton number conservation: we have seen that the brane-bulk mixing relates lepton number on the brane to local $O(2)$ Lorentz transformations in the bulk. Because of this connection it is guaranteed to be a symmetry of the interactions of the 6D supergravity in the bulk. In general this symmetry is spontaneously broken by the background geometry, since extra-dimensional Lorentz rotations are always broken by the zweibein of the internal two dimensions, ${e_m}^a$. For some geometries (such as a 2-sphere) it can happen that the transformation of ${e_m}^a$ under such Lorentz transformations can be compensated by performing a diffeomorphism in the extra dimensions, provided that the extra-dimensional geometry admits a rotational isometry. In such cases the background breaks the product of local Lorentz transformation and diffeomorphism symmetries down to the diagonal rigid rotation which preserves ${e_m}^a$, in an extra-dimensional analogue of the spin-orbit coupling. For geometries without rotational isometries (such as 2-tori) no such compensation is possible and the symmetry is broken completely. This leads to a naturally very small bulk lepton-number breaking scale, of order the KK mass scale, $m_{KK} \sim 0.1\,\mathrm{eV}$. Because of this low symmetry-breaking scale, it is technically natural to have the scale of lepton-number violation in the bulk-brane couplings also be as small as m_{KK}.

Bulk SUSY: in previous studies the properties of the bulk fermions were usually adopted for the convenience of neutrino phenomenology, for lack of a theory of the origin of the bulk fermions, N^I. In MSLED, however, supersymmetry strongly constrains the properties of the bulk. There are, for instance, fermions which are universal to 6D supergravity coming from the supergravity multiplet itself, which typically contain the dilatini, χ, and/or the extra-dimensional components of the gravitino, ψ_m, $m = 4, 5$. If the fermions, N^I, in the bulk-brane mixing terms are linear combinations of these fields, their bulk interactions are strongly constrained by the fact that they are related by supersymmetry to the extra-dimensional graviton. In particular, such fermions cannot have any explicit 6D mass terms at all. Supersymmetry similarly constrains the properties of other bulk fermions, which appear in 6D matter multiplets (hyperini or gaugini), since such fermions are also typically tied by supersymmetry to massless bosons in the bulk.

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2.3 The 4D neutrino mass matrix

We now write down the neutrino bulk-brane mixing which results in a basis of fields for which we assume the charged-lepton masses to already be diagonal. Because no bulk 6D mass terms are allowed by supersymmetry, the complete 4D neutrino mass matrix consists of eq. (2.5) plus the Kaluza-Klein masses coming from the extra-dimensional kinetic terms of the bulk fermions. The result is

$$S_{\nu\,\mathrm{mass}} = -\int d^4x \sum_{\ell I} \left[\frac{\lambda_{aI}^{(+)} v}{r} (\nu_a \gamma_L n_{\ell+}^I) + \frac{\lambda_{aI}^{(-)} v}{r} (\nu_a \gamma_L \tilde{n}_{\ell-}^I) + \frac{c_\ell^I}{r} (n_{\ell+}^I \gamma_L \tilde{n}_{\ell-}^I) + \text{c.c.} \right], \tag{2.8}$$

where $v = 246\,\mathrm{GeV}$ is the Standard Model Higgs *v.e.v.*, and $c_\ell^I \neq 0$ is an $O(2\pi)$ dimensionless number whose value depends on the detailed shape of the extra dimensions and which controls the form of the Kaluza Klein masses through $m_\ell^I = c_\ell^I / r$. For instance for compactifications on a square torus (without brane back-reaction) ℓ is a pair of integers, $\ell = (k_1, k_2)$, and $c_\ell = c_{k_1 k_2} = 2\pi(k_1 + ik_2)$. Here also $\tilde{n}$ denotes the 4D left-handed spinor which is the conjugate to the right-handed spinor n. With (2,0) — or higher — 6D supersymmetry in mind, we do not write an $s = \pm$ dependence for c_ℓ^I because we assume both 2D chiralities to share the same 6D kinetic terms and boundary conditions in the bulk.

Allowing the indices to run over the ranges $a = 1, 2, 3,\ s = \pm$ and $I = 1, \dots, n$, the resulting (left-handed) neutrino mass matrix becomes

$$M_\nu = \frac{1}{r} \begin{pmatrix} 0 & 0 & 0 & g_{11}^{(+)} & g_{11}^{(-)} & g_{12}^{(+)} & g_{12}^{(-)} & g_{13}^{(+)} & g_{13}^{(-)} & \cdots \\ 0 & 0 & 0 & g_{21}^{(+)} & g_{21}^{(-)} & g_{22}^{(+)} & g_{22}^{(-)} & g_{23}^{(+)} & g_{23}^{(-)} & \cdots \\ 0 & 0 & 0 & g_{31}^{(+)} & g_{31}^{(-)} & g_{32}^{(+)} & g_{32}^{(-)} & g_{33}^{(+)} & g_{33}^{(-)} & \cdots \\ g_{11}^{(+)} & g_{21}^{(+)} & g_{31}^{(+)} & 0 & c_\ell^{I=1} & 0 & 0 & 0 & 0 & \cdots \\ g_{11}^{(-)} & g_{21}^{(-)} & g_{31}^{(-)} & c_\ell^{I=1} & 0 & 0 & 0 & 0 & 0 & \cdots \\ g_{12}^{(+)} & g_{22}^{(+)} & g_{32}^{(+)} & 0 & 0 & 0 & c_\ell^{I=2} & 0 & 0 & \cdots \\ g_{12}^{(-)} & g_{22}^{(-)} & g_{32}^{(-)} & 0 & 0 & c_\ell^{I=2} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{2.9}$$

where $g_{aI}^{(s)} = \lambda_{aI}^{(s)} v$. For each ℓ the matrix of diagonal 2×2 blocks with elements c_ℓ^I is a $2n \times 2n$ matrix corresponding to the n bulk fermions, assuming these all share the same boundary conditions (and so also the same KK levels ℓ). We do not require detailed expresions for the c_ℓ^I's, but because of the brane back-reaction we do take all of the $c_\ell^I \neq 0$.

In general this matrix is complex and symmetric, and so may always be diagonalized by finding an appropriate unitary matrix, U_ν, such that

$$U_\nu^T M_\nu U_\nu = M_\nu^{\mathrm{diag}} = \begin{pmatrix} \mu_1 & 0 & 0 & \cdots \\ 0 & \mu_2 & 0 & \cdots \\ 0 & 0 & \mu_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2.10}$$

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where the superscript 'T' denotes transposition (*not* hermitian conjugation). The required U_ν is

$$U_\nu = V_\nu K \tag{2.11}$$

where V_ν is the unitary matrix which diagonalizes the hermitian matrix $M_\nu^\dagger M_\nu$ — but in the usual fashion of a similarity transformation: $V_\nu^\dagger \,(M_\nu^\dagger M_\nu)\, V_\nu = \mathrm{diag}(\mu_1^2, \dots)$ — and K is a diagonal matrix of phases which can be chosen to ensure that the μ_i are non-negative. If the matrix M_ν should be real — as we assume for simplicity below — then V_ν can be chosen more simply to be the usual orthogonal matrix which diagonalizes it. (This diagonalization is performed explicitly for various choices for the mass matrix below.)

Given such a diagonalization, the three species of 3-brane neutrino states may be written as a linear combination of the mass eigenstates according to

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix}, \tag{2.12}$$

where the non-square matrix U consists of the first 3 rows of the matrix U_ν. Putting this into the charged-current interaction, eq. (1.1), shows that it is this matrix, U, which plays the role of the PMNS matrix in extra-dimensional models, and in particular the absence of evidence for sterile neutrinos requires its left-most 3×3 block to agree with the 3×3 PMNS matrix described above, which is inferred from neutrino-oscillation data. Similarly, its 4th and higher columns must all be sufficiently small not to conflict with the various bounds on the existence of sterile neutrinos.

3. Explicit models

In order to proceed further it is necessary to make some choices for the brane-bulk couplings $\lambda_{aI}^{(s)}$, and for the shape of the internal dimensions (which determines the coefficients c_ℓ^I). In this section we consider the simplest choices in some detail, guided by considerations of technical naturalness. For simplicity we assume only a single bulk field couples to the brane, and so henceforth suppress the bulk index I, and so e.g. $\lambda_{aI}^{(s)} \to \lambda_a^{(s)}$.

3.1 A toy test case

Before laying out the couplings which we expect MSLED to provide and which furnish acceptable phenomenology, it is instructive to pause here to examine a simple choice for couplings which does *not* work, in order to identify better the issues which must be addressed by a successful choice. For this purpose consider the choice where all of the couplings are equal, regardless of flavour and chirality: $\lambda_a^{(s)} = \lambda$ for all $a = 1, 2, 3$ and $s = \pm$. As is easily verified, with this choice the mass matrix, eq. (2.9), leads to the following neutrino masses and mass eigenstates:

Massless states: two massless states are given by

$$\nu_0 = \frac{1}{\sqrt{6}}\Big(-2, 1, 1, 0, \cdots\Big)^T \qquad \text{and} \qquad \nu_0' = \frac{1}{\sqrt{2}}\Big(0, 1, -1, 0, \cdots\Big)^T . \tag{3.1}$$

Because the nonzero entries only appear in the first three positions, these massless states do not mix with the infinite tower of bulk modes.

Massive states — part I: for each ℓ there is a neutrino eigenstate having mass $\mu = |m_\ell| = |c_\ell|/r$, given by

$$\nu_k = \frac{1}{\sqrt{2}}\Big(0, 0, 0, \cdots, 0, 1, -1, 0, \cdots\Big)^T , \tag{3.2}$$

where $k = 1, 2, 3, \ldots$ and the two nonzero values appear in the $(2k+2)$'th and $(2k+3)$'th entries of the column vector. These states clearly have the same KK masses as would have been present without the brane-bulk mixing term, and since the first three entries of the eigenstate vanish these states do not mix with the brane neutrinos.

Massive states — part II: all of the other eigenstates also have masses which are nonzero and depend nontrivially on the coupling $g = \lambda v$. These states have masses, $\mu_l = |z_l|/r$, where z_l are defined as the roots of the following equation

$$z = -6g^2 \sum_\ell \frac{1}{c_\ell - z} . \tag{3.3}$$

The corresponding eigenstates are given by:

$$\nu_l = N_l \left(-\frac{1}{3g}, -\frac{1}{3g}, -\frac{1}{3g}, \frac{1}{c_{\ell=1} - z_l}, \frac{1}{c_{\ell=1} - z_l}, \frac{1}{c_{\ell=2} - z_l}, \frac{1}{c_{\ell=2} - z_l}, \cdots \right)^T , \tag{3.4}$$

with normalization constant N_l.

It is instructive to ask what happens to these states in the limit $g \to 0$. As may be seen, for instance by solving this equation graphically, all but one of the μ_l's approaches the KK masses, $|m_\ell| = |c_\ell|/r$ in this limit. The one state whose mass does not approach $|m_\ell|$ as $g \to 0$ has a mass, μ_0, which is bounded above by

$$\mu_0 \leq \frac{6g^2}{r} \, |\mathcal{S}| , \tag{3.5}$$

where

$$\mathcal{S} \equiv \sum_\ell \frac{1}{c_\ell} . \tag{3.6}$$

Evaluating μ_0 perturbatively in g shows that μ_0 actually equals this upper bound up to corrections which are $O(g^4)$.

Although this mass nominally vanishes as $g \to 0$, what is interesting is that the sum appearing in μ_0 generically diverges in the UV, with a divergence linear in the cutoff

$L \sim M_g/m_{KK} \sim M_g r/(2\pi) \sim 10^{14}$ for two extra dimensions — where the numerical estimate takes $M_g \sim 10\,\mathrm{TeV}$ and $m_{KK} \sim 0.1\,\mathrm{eV}$. Keeping track of factors of 2π, and recalling $c_\ell \sim 2\pi$, we take $\mathcal{S} \sim \frac{1}{2\pi}(2\pi L) = L$. This illustrates how extra-dimensional models can complicate the problem of identifying masses and mixings in the small-g limit, because the divergences associated with sums over the very large number of KK modes (like $\mathcal{S}$) can overpower small pre-multiplying factors (like g^2), even for the very small values $g^2/(2\pi) \lesssim 10^{-8}$ of interest in what follows.

The divergence appearing in μ_0 may be absorbed as a renormalization of a counter-term localized on the brane, which has the form of the usual Standard Model dimension-5 neutrino mass:

$$S_{\rm brane} = h_{ab} \int d^4x\, (L_a H)(L_b H)\,. \tag{3.7}$$

The required renormalization is of order $\delta h \sim \lambda^2 M_g \sim 1/M_g$. Since the neutrino masses induced by such an interaction are of order hv^2 and the natural scale for the h_{ab} is of order $1/M_g$, the existence of such a term underlines the fact that any successful explanation of neutrino masses using brane-bulk couplings within MSLED must also explain the *absence* of this pure brane term, which can produce much too large a mass [14].

A second type of divergent sum which we shall also encounter is $g^2\,\mathcal{P}$, where

$$\mathcal{P} = \sum_\ell \frac{1}{c_\ell^2}\,. \tag{3.8}$$

Because this diverges logarithmically for two extra dimensions — $\mathcal{P} \sim \frac{1}{(2\pi)^2}(2\pi \ln L) \sim \frac{1}{2\pi}\ln(M_g/m_{KK}) \sim 32/(2\pi)$ — the combination $g^2\mathcal{P}$ can be large provided $g^2/(2\pi) \gtrsim 0.03$. In practice, our interest in the regime $g \lesssim 10^{-4}$ allows us to neglect such sums, but we nevertheless provide in an appendix expressions for the eigenvalues and eigenvectors which do not assume the quantity $g^2\mathcal{P}$ to be small.

3.2 Perturbative eigenvalues and eigenvectors

As the above example shows, agreement with the neutrino data requires more complicated bulk-brane couplings than those which were entertained for the Toy Model. Before turning to specific proposals for more phenomenologically successful brane-bulk couplings, we pause here to record general expressions for the eigenvalues and eigenvectors for more complicated neutrino mass matrices. Keeping in mind the strong bounds on active-sterile mixing, and in order to keep the analysis simple, we solve the eigenvalue and eigenvector problem to leading nontrivial order in the brane-bulk couplings, after first stating some exact definitions. (We also provide a more detailed calculations for some phenomenologically interesting choices for the mass matrix in the appendix.)

Our interest is in diagonalizing the mass matrix of eq. (2.9), in the special case of a single bulk field but with couplings $\lambda_a^{(\pm)}$ which are otherwise arbitrary. To this end we write the mass eigenstate corresponding to the lowest three mass eigenvalues, $\mu_i = \mu_{0,\pm} = z_i/r = O(g^2/r)$ as follows:

$$\nu_i = N_i\Big(n_i e_{1i}, n_i e_{2i}, n_i e_{3i}, x_{1i}, y_{1i}, \dots, x_{\ell i}, y_{\ell i}, \dots\Big)^T, \tag{3.9}$$

where N_i is an overall normalization constant and n_i is chosen to ensure that the e_{ai} satisfy $e^2_{1i} + e^2_{2i} + e^2_{3i} = 1$. The normalization constant is given by $N_i = [n_i^2 + \Sigma_i^2]^{-1/2}$ where Σ_i denotes the sum

$$\Sigma_i^2 = \sum_\ell (x^2_{\ell i} + y^2_{\ell i})\,. \tag{3.10}$$

In analyzing the physical implications of any such an eigenstate it is convenient to define the active-sterile mixing angle by $c_{si} = \cos\theta_{si} = N_i\, n_i$, so that the first three entries for ν_i become $c_{si}\,\vec{e}_i$, with $\vec{e}_i \cdot \vec{e}_i = 1$. In terms of Σ_i this is may be written in the useful equivalent form

$$\tan\theta_{si} = \frac{\Sigma_i}{n_i}\,. \tag{3.11}$$

Notice that the unitarity of the full matrix U_{ai}, together with the unitarity of the 3×3 matrix whose elements are e_{ai}, implies that this angle is the one which is relevant to the total incoherent energy loss into sterile neutrinos from a charged-current reaction involving lepton flavour ℓ_a:

$$R_a = \sum_{i>3} |U_{ai}|^2 = 1 - \sum_{i=1}^{3} |U_{ai}|^2 = 1 - \sum_{i=1}^{3} c^2_{si}\,|e_{ai}|^2 = \sum_{i=1}^{3} s^2_{si}\,|e_{ai}|^2\,. \tag{3.12}$$

Writing, as before, $g_a^{(\pm)} = \lambda_a^{(\pm)} v$, it is fairly simple to compute the mass eigenvalues and eigenstates to second order in the small quantities $g_a^{(\pm)}$. Within perturbation theory, the leading-order masses, μ_i, and the components, e_{ai}, for the lightest three states are determined by diagonalizing the following perturbative correction to the mass-matrix within the degenerate 3-dimensional massless subspace:

$$\mu_{ab} = -\frac{\mathcal{S}}{r}\left(g_a^{(+)} g_b^{(-)} + g_b^{(+)} g_a^{(-)}\right). \tag{3.13}$$

This matrix has rank 2, which ensures that one of the three light neutrinos remains massless to this order. Once the components e_{ai}, are obtained in this way, the leading expressions for the components $x_{\ell i}$ and $y_{\ell i}$ are similarly given by the perturbative expressions

$$x_{\ell i} = -\frac{\vec{e}_i \cdot \vec{g}^{(-)}}{c_\ell} \quad \text{and} \quad y_{\ell i} = -\frac{\vec{e}_i \cdot \vec{g}^{(+)}}{c_\ell}\,, \tag{3.14}$$

where $\vec{e}_i \cdot \vec{g}^{(\pm)} = \sum_{a=1}^{3} e_{ai}\, g_a^{(\pm)}$. Similarly, the quantity $n_i = 1$ up to corrections which are 2nd order in the $g^{(\pm)}$. To leading order the active-sterile mixing angle, θ_{si}, for the three neutrino species whose mass vanishes as $g \to 0$ then becomes

$$\tan^2\theta_{si} \approx \Sigma_i^2 = \sum_\ell \left(x^2_{\ell i} + y^2_{\ell i}\right) = \left[(\vec{e}_i \cdot \vec{g}^{(+)})^2 + (\vec{e}_i \cdot \vec{g}^{(-)})^2\right]\mathcal{P}\,, \tag{3.15}$$

where $\mathcal{P}$ is as in eq. (3.8).

3.3 Approximate symmetries I

With these insights in mind, we next explore a more realistic model of neutrino masses within the SLED framework. We start in this section with a model having a minimal flavour content, which can satisfy all of the required phenomenological constraints but one. Even though it does not succeed in the end, we describe this model in detail here for several reasons. First, we do so because it comes so close and illustrates well the tension between the various constraints which makes constructing a successful model difficult. Second, it is useful to describe in detail the symmetry issues which the model raises, since these also play an important role in the successful model we describe in the next section.

The simplest and most natural choice to make for the brane-bulk couplings at the TeV scale is to suppose them to be both flavour-independent and Lepton-number conserving. Lepton number conservation is necessary because it precludes the appearance of a brane neutrino mass like eq. (3.7). Flavour-independence is a natural expectation since couplings to the bulk are gravitational, and so the microscopic physics which underlies them is unlikely to care about the details of the low-energy flavour physics on the brane.

These symmetries suggest we take the couplings at the TeV scale to be

$$\lambda_a^{(+)} = 0 \qquad \text{and} \qquad \lambda_a^{(-)} = \lambda\,, \tag{3.16}$$

for all brane flavours, a, which preserves the Standard-Model lepton number, $L = L_e + L_\mu + L_\tau$, in addition to the extra-dimensional chirality, J. With this choice there are precisely 3 massless neutrino eigenstates as well as a tower of KK modes whose Dirac masses are nonzero. The masslessness of the three lightest states is protected by the lepton number invariance. Notice that because these states are degenerate in the leading approximation, their mutual mixings need *not* be small once this mass matrix is perturbed to obtain more accurately the neutrino masses and eigenstates. We do not have more than 3 such degenerate states — unlike previous analyses — because of our inclusion of the brane back-reaction, which removes any KK zero modes.

Now we imagine renormalizing these couplings down to the lower energies relevant to neutrino physics, and we expect that neither the flavour nor the lepton-number invariances are likely to survive this process as an exact symmetry of the low-energy theory. Flavour symmetry is broken at the very least because the Standard Model brane couplings themselves distinguish amongst flavours. Similarly, we have seen that within the bulk the role of lepton number is carried by local Lorentz rotations, J, in the extra dimensions, and these are naturally broken by the background extra-dimensional geometry, such as by the background zweibein, ${e_m}^a$.[5] The symmetry-breaking scale for this symmetry is then naturally of order the KK scale, $m_{KK} \sim 2\pi/r$. Once such corrections are included into the

[5]Notice that ${e_m}^a$ can remain invariant if the effects of the local Lorentz transformation acting on a is cancelled by performing a compensating diffeomorphism on the index m. The diagonal transformation which preserves the zweibein can then be an unbroken symmetry of the full theory if the compensating diffeomorphism is also an isometry of the metric $g_{mn} = {e_m}^a\, {e_n}^b\, \delta_{ab}$. For instance, it can be unbroken if the internal metric is rotationally-invariant (like for the 2-sphere) but would be broken if not (like for the 2-torus).

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brane-bulk couplings we expect eq. (3.16) to be replaced by

$$g_a^{(+)} = \epsilon_a^{(+)} \qquad \text{and} \qquad g_a^{(-)} = g + \epsilon_a^{(-)} \,, \tag{3.17}$$

where as before $g = \lambda v$, and the $\epsilon_a^{(\pm)}$ are of order $m_{KK}/M_J \sim K m_{KK}/M_g \sim 2\pi\, K/(M_g r) \ll g \sim v/M_g$, where M_J denotes the mass scale which communicates the symmetry-breaking scale to the brane sector, and $K = M_g/M_J$ is a dimensionless constant which we choose to obtain an acceptable neutrino phenomenology.[6] Our task now is to infer the neutrino masses and mixing which are implied by such a coupling choice.

To second order in $g_a^{(\pm)}$ the effective mass matrix, eq. (3.13), which determines the mass shift of the three massless unperturbed states becomes

$$\mu_{ab} \approx -\frac{g\,\mathcal{S}}{r}\left(\epsilon_a^{(+)} + \epsilon_b^{(+)}\right) , \tag{3.18}$$

where the approximate equality drops terms which are of order ϵ^2 in comparison with those which are order ϵg.

This matrix has rank 2, which ensures that one of the three light neutrinos remains massless to this order. The other two neutrinos acquire masses which are generically of order $\epsilon g/r$ times the divergent sum $\mathcal{S}$. Keeping in mind that $\epsilon \sim K m_{KK}/M_g$ and the sum diverges linearly in the cutoff, which is of order $L \sim M_g/m_{KK}$, we see that the nonzero masses which are generated are of order $\epsilon g\, L/r \sim gK/r$. Because of the protection of the approximate lepton number symmetry — i.e. the small size of $\epsilon_a^{(s)}$ — these are the right order of magnitude to describe the observed neutrino masses.[7]

Obtaining the precise form for the masses and mixings of the lightest three states requires making more specific choices for the $\epsilon_a^{(+)}$'s, and guided by neutrino-oscillation phenomenology described in earlier sections we choose these to preserve the Z_2 symmetry which interchanges the 2nd and 3rd generations. In particular we take

$$\epsilon_a^{(+)} = (\epsilon_1, \epsilon_2, \epsilon_2)^T \,, \tag{3.19}$$

in which case the one massless eigenvector is the standard one,

$$\nu_0 = \frac{1}{\sqrt{2}}\,(0, 1, -1, 0, \dots)^T \,, \tag{3.20}$$

for which $x_{\ell 0} = y_{\ell 0} = \sin\theta_{s0} = 0$. The two nonzero eigenvalues which vanish as $g \to 0$ become

$$\mu_\pm = \frac{g\,\mathcal{S}}{r}\,\left[(\epsilon_1 + 2\epsilon_2) \pm \left((\epsilon_1 + 2\epsilon_2)^2 + 2(\epsilon_1 - \epsilon_2)^2\right)^{1/2}\right] , \tag{3.21}$$

[6] As discussed in ref. [10], an explicit microscopic realization of the SLED picture is likely to involve a number of mass scales smaller than the 6D Planck constant, M_g, such as the string scale, M_s, and the KK scale for any dimensions in addition to the 6 discussed here.

[7] To the extent that bounds on sterile neutrinos require us to take $g \lesssim 10^{-4}$ we must imagine $K \sim 10^4$. An explanation for why K should be so large is a requirement for any explicit SLED compactification which hopes to describe neutrino phenomenology. We here content ourselves with a phenomenological approach, showing that neutrino phenomenology need not conflict with sterile neutrino bounds in SLED models. We leave for future work the explicit calculation of symmetry-breaking effects like K in an explicit model.

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where $\mathcal{P}$ is as defined in eq. (3.8). Notice the $\mu_\pm$ are $O(\epsilon_i g\mathcal{S}/r)$, as required. The corresponding neutrino eigenstates have

$$\vec{e}_\pm = \frac{1}{\sqrt{2+\alpha_\pm^2}}(\alpha_\pm, 1, 1)^T \,, \tag{3.22}$$

where

$$\alpha_\pm = \frac{(\epsilon_1 - 2\epsilon_2) \pm \Big((\epsilon_1+2\epsilon_2)^2 + 2(\epsilon_1-\epsilon_2)^2\Big)^{1/2}}{\epsilon_1+\epsilon_2} \,. \tag{3.23}$$

It follows that $\theta_{s\pm}$ is given by

$$\tan^2\theta_{s\pm} \approx \left[\frac{(\alpha_\pm+2)^2}{\alpha_\pm^2+2}\right] g^2\mathcal{P} \,, \tag{3.24}$$

where the approximate equality uses $\epsilon_i \ll g$.

Notice that if $\epsilon_1 = \epsilon_2 \equiv \epsilon$, then $\alpha_+ = 1$ and $\alpha_- = -2$, and so $\tan^2\theta_{s+} = 3g^2\mathcal{P}$ and $\tan^2\theta_{s-} = 0$. This leads to the following PMNS matrix for the lightest three neutrino states

$$U \approx \begin{pmatrix} -\sqrt{2/3} & c_{s+}/\sqrt{3} & 0 \\ 1/\sqrt{6} & c_{s+}/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & c_{s+}/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \,, \tag{3.25}$$

whose matrix elements agree (in the limit $\theta_{s+} \to 0$) in magnitude with the tri-bimaximal mixing matrix described earlier. For these same choices the neutrino mass values of eq. (3.21) become $\mu_- = 0$ and $\mu_+ = 6\epsilon g\mathcal{S}/r$.

The problem with this model is that it does not appear to have a parameter range for which the neutrino mass spectrum, mixing angles *and* sterile-active mixing angle are all acceptable for the same choice of parameters. To see this notice that if we imagine taking g sufficiently small to minimize the active-sterile mixing, then we expect the observed neutrino oscillations to be obtained from the mixings of the three states whose masses vanish in the limit $g \to 0$. Since our conventions assume that for three-neutrino mixing it is ν_1 and ν_2 which dominantly participate in solar-neutrino oscillations, we see that the observed hierarchy $|\Delta m^2_\odot| \ll |\Delta m^2_{\rm atm}|$ then requires us to take $(\mu_+ - \mu_-)^2 \ll |\mu_+\mu_-|$, and so $|\epsilon_1 + 2\epsilon_2| \ll 2|\epsilon_1 - \epsilon_2|$. Notice that this condition is precisely the opposite to the choice $\epsilon_1 = \epsilon_2$ which led to the successful PMNS mixing matrix, above.

Although we have not found a choice for the ϵ_i which produces acceptable masses and mixings using the perturbative analysis just described, we *can* do so if we make g large enough that the quantity $g^2\mathcal{P}$ is not small. In this case the more exact diagonalization of the appendix shows that eq. (3.21) generalizes to $\mu_\pm = |z_\pm|/r$, with

$$z_\pm = g\,\mathcal{S}\left[\frac{(\epsilon_1+2\epsilon_2) \pm \Big((\epsilon_1+2\epsilon_2)^2 + 2(\epsilon_1-\epsilon_2)^2(1+3g^2\mathcal{P})\Big)^{1/2}}{1+3g^2\mathcal{P}}\right] \,, \tag{3.26}$$

and so the condition for obtaining a small $\nu_1 - \nu_2$ mass splitting becomes

$$(\epsilon_1+2\epsilon_2)^2 \ll 2(\epsilon_1-\epsilon_2)^2(1+3g^2\mathcal{P}) \,. \tag{3.27}$$

When $g^2\mathcal{P} \gtrsim 1$ the quantities which control the mixing angles become

$$\alpha_i = \frac{2[(g^2+\epsilon_1\epsilon_2)z_i\hat{\mathcal{P}}(z_i) + g(\epsilon_1+\epsilon_2)\hat{\mathcal{S}}(z_i)]}{z_i - (g^2+\epsilon_1^2)z_i\hat{\mathcal{P}}(z_i) - 2g\epsilon_1\hat{\mathcal{S}}(z_i)} \approx \frac{2[g^2 z_i\hat{\mathcal{P}}(z_i) + g(\epsilon_1+\epsilon_2)\hat{\mathcal{S}}(z_i)]}{z_i - g^2 z_i\hat{\mathcal{P}}(z_i) - 2g\epsilon_1\hat{\mathcal{S}}(z_i)} \tag{3.28}$$

and

$$\begin{aligned} x_{\ell i} &= \frac{g(\alpha_i+2)c_\ell + (\alpha_i\epsilon_1 + 2\epsilon_2)z_i}{z_i^2 - c_\ell^2} \approx \frac{g(\alpha_i+2)c_\ell}{z_i^2 - c_\ell^2} \\ y_{\ell i} &= \frac{g(\alpha_i+2)z_i + (\alpha_i\epsilon_1 + 2\epsilon_2)c_\ell}{z_i^2 - c_\ell^2} \approx \frac{g(\alpha_i+2)z_i}{z_i^2 - c_\ell^2}\,, \end{aligned} \tag{3.29}$$

where the approximate equality for $y_{\ell i}$ applies for all ℓ if z_i is not too small, but is not valid for ℓ too close to the cutoff if $z_i \sim O(1)$. Here, as before, $|z_i| = \mu_i r$, and the functions $\hat{\mathcal{S}}(z)$ and $\hat{\mathcal{P}}(z)$ are defined by the sums

$$\hat{\mathcal{S}}(z) = \sum_\ell \frac{c_\ell}{z^2 - c_\ell^2} \qquad \text{and} \qquad \hat{\mathcal{P}}(z) = \sum_\ell \frac{1}{z^2 - c_\ell^2} \tag{3.30}$$

(from which we see $\hat{\mathcal{S}}(0) = -\mathcal{S}$ and $\hat{\mathcal{P}}(0) = -\mathcal{P}$, and so in particular $\epsilon_i g\hat{\mathcal{S}}(z) \sim O(1)$). Finally, assuming the sum over $x_{\ell i}^2 + y_{\ell i}^2$ to be dominated by its UV divergent part, we have

$$\sin^2\theta_{si} \approx \left[\frac{(\alpha_i+2)^2}{\alpha_i^2+2}\right] g^2\mathcal{P}\,. \tag{3.31}$$

From these expressions we see that μ_+ and μ_- can be made sufficiently degenerate without giving up an acceptable PMNS mixing matrix, e_{ai}, provided we take $g^2\mathcal{P}$ to be sufficiently large. (Recall that $\mathcal{P} \sim 32$ and so $g^2\mathcal{P}$ can be taken to be large while still keeping g small enough to not lose perturbative control over loops involving g.) However the penalty paid in this case is unacceptably large active-sterile mixing angles, θ_{si}. Although we have not been able to find a choice of parameters within this class which satisfies all observational constraints, neither have we been able to completely exclude that this is possible.

3.4 Approximate symmetries II

We next turn to a pattern of lepton and flavour symmetries which can produce acceptable neutrino phenomenology. We present this model in the spirit of an existence proof that such models can be possible within the SLED framework.

As before we must demand brane-bulk neutrino couplings which conserve a lepton number in order not to generate too large neutrino masses as we integrate out scales between the TeV scale and the sub-eV scale. In this case, however, we assume that our couplings at the TeV scale separately conserve the lepton numbers L_e and $L_\mu + L_\tau$, with couplings

$$\lambda_a^{(+)} = \Big(0, \lambda, \lambda\Big)^T \qquad \text{and} \qquad \lambda_a^{(-)} = 0\,. \tag{3.32}$$

With this choice all three neutrino masses vanish, and the masslessness of the three lightest states is protected by the lepton number invariance.

We next assume that the renormalization down to the sub-eV scale breaks lepton number, but in such a way that the combination $L_e - L_\mu - L_\tau$ is less badly broken than the other combinations, in which case

$$g_a^{(+)} = \lambda_a^{(+)} v = \Big(0, g, g\Big)^T \qquad \text{and} \qquad g_a^{(-)} = \lambda_a^{(-)} v = \Big(\epsilon, 0, 0\Big)^T . \tag{3.33}$$

As before we take $\epsilon \sim K m_{KK}/M_g \ll g \ll 1$, and so the lightest nonzero neutrino masses are of order $\epsilon g \mathcal{S}/r$. For simplicity we also assume here the permutation symmetry which interchanges the 2nd and 3rd generations, although perturbations about this limit are permissible provided they do not move the atmospheric neutrino oscillations too far away from maximal mixing.

With these choices we have the usual massless mass eigenstate

$$\nu_0 = \frac{1}{\sqrt{2}} \, (0, 1, -1, 0, \cdots)^T , \tag{3.34}$$

for which $x_{0j} = y_{0j} = \sin\theta_{s0} = 0$. In this case the unbroken $L_e - L_\mu - L_\tau$ symmetry ensures that the two nonzero eigenvalues which vanish as $g \to 0$ form a Dirac pair with lowest-order mass

$$\mu^0_\pm = \frac{\sqrt{2}\, \epsilon g \mathcal{S}}{r} , \tag{3.35}$$

and eigenvectors

$$\nu_\pm = \left(\pm\frac{c_{s\pm}}{\sqrt{2}}, \frac{c_{s\pm}}{2}, \frac{c_{s\pm}}{2}, x_{1\pm}, y_{1\pm}, \dots x_{\ell\pm}, y_{\ell\pm}, \dots \right)^T \tag{3.36}$$

to leading order in g. These expressions use the lowest-order result $\alpha^0_\pm = \pm\sqrt{2}$ and

$$x_{\ell\pm} = \mp \frac{\epsilon}{\sqrt{2} c_\ell} \quad \text{and} \quad y_{\ell\pm} = -\frac{g}{c_\ell} , \tag{3.37}$$

and so $\theta_{s+} = \theta_{s-} = \theta_s$, where as before

$$\tan^2\theta_s \approx g^2 \mathcal{P} . \tag{3.38}$$

Phenomenologically acceptable masses and mixings are obtained by perturbing away from this limit, replacing eqs. (3.33) with

$$g_a^{(+)} = \lambda_a^{(+)} v = \big(-2g', g, g\big)^T \qquad \text{and} \qquad g_a^{(-)} = \lambda_a^{(-)} v = \big(\epsilon, \epsilon', \epsilon'\big)^T , \tag{3.39}$$

where g'/g and ϵ'/ϵ are both 10%, but $g'/g - \epsilon'/\epsilon \lesssim 1\%$. This leads to the lowest-order mass matrix

$$\mu_{ab} = -\frac{\mathcal{S}}{r} \left(g_a^{(+)} g_b^{(-)} + g_a^{(-)} g_b^{(+)} \right) \approx -\frac{\mathcal{S}}{r} \begin{pmatrix} -4\epsilon g' & \epsilon g - 2\epsilon' g' & \epsilon g - 2\epsilon' g' \\ \epsilon g - 2\epsilon' g' & 2g\epsilon' & 2g\epsilon' \\ \epsilon g - 2\epsilon' g' & 2g\epsilon' & 2g\epsilon' \end{pmatrix} , \tag{3.40}$$

whose properties we evaluate perturbatively in g'/g and ϵ'/ϵ.

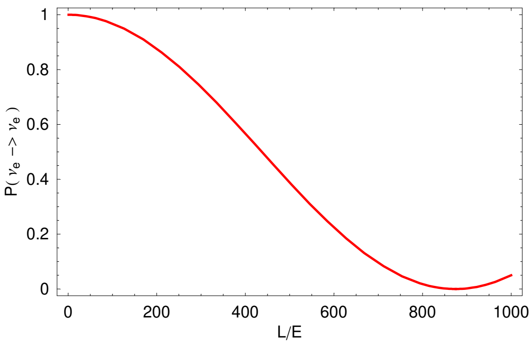

Figure 1: The ν_e survival probability as a function of L/E in m/MeV, using $\epsilon'/\epsilon = 0.1$, $g'/g = 0.11$, $g = 10^{-3}$, $\mathcal{P} = 32$, $1/r = 0.01\,\text{eV}$ and $\mu^0_\pm = 0.05\,\text{eV}$.

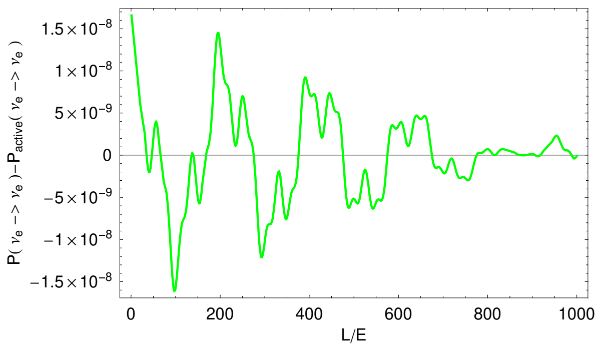

Figure 2: The difference between the full survival probability and the survival probability computed only using the 'active' states ν_0 and $\nu_\pm$.

With these choices the massless state is not perturbed: $\delta\mu_0 = 0$ and $\delta\nu_0 = 0$. On the other hand, including the leading and next-to-leading shift in the lightest (but nonzero) two perturbed mass eigenvalues gives

$$\mu_\pm = \mu^0_\pm \left[1 \pm \sqrt{2} \left(\frac{\epsilon'}{\epsilon} - \frac{g'}{g} \right) + \left(\frac{\epsilon'}{\epsilon} \right)^2 + \left(\frac{g'}{g} \right)^2 + \cdots \right], \tag{3.41}$$

where $\mu^0_\pm$ is given in eq. (3.35). We work to next-to-leading order in this expression since our assumptions imply that the difference $g'/g - \epsilon'/\epsilon$ is the same order as $(g'/g)^2$ and $(\epsilon'/\epsilon)^2$.

The corrected values for $\alpha_\pm$ are $\alpha_\pm = \alpha^0_\pm - \delta + \cdots$ where $\alpha^0_\pm = \pm\sqrt{2}$ and

$$\delta = 2 \left(\frac{\epsilon'}{\epsilon} + \frac{g'}{g} \right), \tag{3.42}$$

and so the mixing matrix for the lightest three neutrinos becomes to this order

$$U \approx \begin{pmatrix} c_s(-1/\sqrt{2} - \delta/4) & c_s(1/\sqrt{2} - \delta/4) & 0 \\ c_s(1/2 - \delta/4\sqrt{2}) & c_s(1/2 + \delta/4\sqrt{2}) & 1/\sqrt{2} \\ c_s(1/2 - \delta/4\sqrt{2}) & c_s(1/2 + \delta/4\sqrt{2}) & -1/\sqrt{2} \end{pmatrix}. \tag{3.43}$$

Figure 1 shows the survival probability for vacuum oscillations, computed by numerically performing the mode sum in the expression

$$P(\nu_a \to \nu_a) = \sum_i |U_{ai}|^4 + 2 \sum_{i>j} |U_{ai}|^2 |U_{aj}|^2 \cos\left(\frac{\Delta m^2_{ij} L}{2E} \right), \tag{3.44}$$

using a representative set of parameter values and the KK spectrum of a square torus (with the zero mode removed). By contrast, figure 2 shows the contribution of the sterile neutrinos to the survival probability. This shows that the oscillations are well described by oscillations among the usual three active flavours, with the sterile oscillations only introducing an unobservably small 'fine structure'. Furthermore, the active mixing angles given by eq. (3.43) can describe the data when g'/g and ϵ'/ϵ are approximately 10% in size, because a 10% shift is sufficient to make the solar mixing angle acceptably far from maximal mixing. Yet the ratio of mass splittings relevant to solar and atmospheric neutrinos is also acceptable because

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm atm}} = \frac{(\mu_+^2 - \mu_-^2)}{\left[\frac{1}{2}(\mu_\pm + \mu_-)\right]^2} = \frac{4(\mu_+ - \mu_-)}{\mu_+ + \mu_-} \approx 8\sqrt{2}\left(\frac{\epsilon'}{\epsilon} - \frac{g'}{g}\right) \sim 1\%\,. \qquad (3.45)$$

Because of the factor of $8\sqrt{2}$ this works provided the difference $\epsilon'/\epsilon - g'/g$ is about 1% of the value of each of ϵ'/ϵ and g'/g separately, which unlike the other choices we have made does not seem to follow purely from symmetry grounds.

Active-sterile mixings are kept acceptable in this model by taking $g \lesssim 10^{-4}$, and this in turn requires that the constant K appearing in $\epsilon = K m_{KK}/M_g$ is of order 10^4 so that the mass scales are $\mu_\pm \sim \epsilon g \mathcal{S}/r \sim Kg/r \sim 1/r \sim 0.01\,\mathrm{eV}$, as required. Since $K = M_g/M_J$, where M_J is the mass scale of the sector which communicates the bulk lepton-number breaking to the brane, taking $M_g = 10\,\mathrm{TeV}$ implies the worrisomely-low value $M_J \sim 1\,\mathrm{GeV}$. The question whether these choices for K, ϵ' and g' can be obtained from a simple extra-dimensional geometry is presently under study.

4. Conclusions

The coincidence between the values of the recently-discovered neutrino masses and the Dark Energy density suggests the possibility of a deep connection between these quantities — a possibility which can be more sharply posed within the SLED proposal for resolving the cosmological constant problem. Embedding extra-dimensional neutrino models into SLED opens up many attractive features, largely because the supersymmetry of the bulk physics necessarily constrains the otherwise arbitrary properties of the bulk sterile neutrinos. In particular, it removes the need for the *ad hoc* assumption of vanishing bulk fermion masses, since supersymmetry requires these masses to vanish. As is also true for non-supersymmetric models, brane-bulk neutrino mixing provides a natural extension of lepton number to bulk fermions, by embedding it within local Lorentz rotations of the large extra dimensions.

In this paper we provide an example of a model which realizes this connection in a concrete way, thereby motivating a more detailed examination of neutrino models within the SLED context. The possibility of so doing comes as a surprise, since past experience of neutrino models involving large extra dimensions were driven by sterile-neutrino constraints towards extra dimensions having properties which are inconsistent with the requirements of the SLED picture for the cosmological constant problem. In particular, constraints on catastrophic neutrino emission into the bulk typically drive model builders to choose only one dimension to exist near the eV scale, since otherwise the higher-dimensional phase space can make this emission rate much too large.

By contrast, we here find that astrophysical and cosmological bulk-neutrino emission can be acceptably small provided that the brane-bulk mixing is itself small ($g/2\pi \lesssim 10^{-4}$). The somewhat surprising result that this is possible follows largely from the observation

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that back-reaction of the stress-energy of the brane typically removes the massless fermion KK mode. (For example, many co-dimension 2 branes induce a conical singularity in the extra-dimensional geometry at the brane position and the boundary conditions which this conical defect imposes typically removes the massless bulk fermion modes.) The removal of these zero modes is crucial for successful neutrino phenomenology, since even a small brane-bulk couplings typically induces large mixings amongst any degenerate massless bulk and brane states. If no bulk zero modes exist then active-sterile neutrino mixings remain small for small brane-bulk couplings, and the large mixings amongst the active neutrinos can explain the observed neutrino oscillation pattern.

We construct a model having acceptable phenomenology by assuming a relatively mild flavour-dependence for the brane-bulk couplings, and by taking these couplings to be sufficiently small. Technically natural small neutrino masses are obtained, with approximate lepton number protecting the small masses of the lightest neutrinos. This is possible because lepton number extends into the extra dimensions as a local Lorentz transformation, and for generic internal geometries it is naturally broken at the Kaluza-Klein scale. A potentially unpleasant feature of the small brane-bulk couplings which we assume is that the amount of lepton number breaking which is necessary to get acceptable neutrino oscillations appears to require a relatively large coupling of bulk lepton-number breaking to the brane neutrinos.

Although it is not yet clear whether such couplings can be generated from real compactifications, we believe the success of the models described herein to motivate more detailed studies of the possibilities offered by neutrino model building within the SLED framework.

Acknowledgments

We thank Marco Cirelli, George Fuller and David London for helpful discussions. C.B.'s research is supported by a grant from NSERC (Canada), McMaster University and the Killam foundation and J.M. receives funds from the Ramon y Cajal Program, FPA2002-00748 and PNL2005-41. C.B. thanks the hospitality of the Aspen Center for Physics, where this paper was finalized.

A. Diagonalization for large mixings

In this appendix we present the exact results for the eigenvectors and eigenvalues for the mass matrix in the two cases discussed in the main text.

A.1 Approximate symmetries I

We take here the symmetry choice of section 3.3 and impose $\epsilon_i^- = 0$ for simplicity, implying

$$g^{(+)} = (\epsilon_1, \epsilon_2, \epsilon_2)^T \quad g^{(-)} = (g, g, g)^T$$

The eigenvectors and eigenstates for the mass matrix in this case come in two distinct kinds:

- There is an eigenstate whose eigenvalue is exactly zero:

$$\nu_0 = \frac{1}{\sqrt{2}} (0, 1, -1, 0, 0, 0, 0, \ldots)$$

- All other eigenstates have nonzero eigenvalues given by $\mu_i = |z_i|/r$, where the quantity z_i is the solution of the following transcendental equation

$$\left(-2g\mathcal{S}'_i\epsilon_1 + \left[1 - \mathcal{P}'_i \left(g^2 + \epsilon_1^2\right)\right] z_i\right)\left(-4g\mathcal{S}'_i\epsilon_2 + \left[1 - 2\mathcal{P}'_i \left(g^2 + \epsilon_2^2\right)\right] z_i\right) - \\ -2\Big(g\mathcal{S}'_i(\epsilon_1 + \epsilon_2) + \mathcal{P}'_i(g^2 + \epsilon_1\epsilon_2)z_i\Big)^2 = 0\,, \quad \text{(A.1)}$$

where

$$\mathcal{S}'_i = \sum_\ell S_{\ell i} \quad \text{with} \quad S'_{\ell i} \equiv \frac{c_\ell}{z_i^2 - c_\ell^2} \quad \text{and} \quad \mathcal{P}'_i = \sum_\ell P'_{\ell i} \quad \text{with} \quad P'_{\ell i} \equiv \frac{1}{z_i^2 - c_\ell^2}\,. \quad \text{(A.2)}$$

The corresponding eigenvectors are

$$\nu_i = N_i \left(\alpha_i, 1, 1, x_{1i}, y_{1i}, x_{2i}, y_{2i}, x_{3i}, y_{3i}, \ldots\right)\,, \quad \text{(A.3)}$$

where

$$\begin{aligned} \alpha_i &= \frac{2g\mathcal{S}'_i \left(\epsilon_1 + \epsilon_2\right) + 2\mathcal{P}'_i z_i \left(g^2 + \epsilon_1\epsilon_2\right)}{z_i \left(1 - (g^2 + \epsilon_1^2)\mathcal{P}'_i\right) - 2g\epsilon_1\mathcal{S}'_i}\,, \\ x_{\ell i} &= gS'_{\ell i}\left(\alpha_i + 2\right) + z_i P'_{\ell i}\left(2\epsilon_2 + \alpha_i\epsilon_1\right)\,, \\ y_{\ell i} &= S'_{\ell i}\left(2\epsilon_2 + \alpha_i\epsilon_1\right) + g z_i P'_{\ell i}\left(\alpha_i + 2\right) \end{aligned} \quad \text{(A.4)}$$

and the overall normalization factor is given in terms of these by

$$N_i = \frac{1}{\sqrt{2 + \alpha_i^2 + \sum_j \left(x_{ji}^2 + y_{ji}^2\right)}}\,. \quad \text{(A.5)}$$

The solutions to the eigenvalue equation (A.1) come in the following two different types:

- Two light eigenvalues that in the limit of $z_\pm \ll c_\ell^2$ are:

$$z_\pm = -g\mathcal{S}\frac{\left[(\epsilon_1 + 2\epsilon_2) \pm \left((\epsilon_1 + 2\epsilon_2)^2 + 2(\epsilon_1 - \epsilon_2)^2 u_i\right)^{1/2}\right]}{u_i} \quad \text{(A.6)}$$

where

$$u_i = 1 + (3g^2 + \epsilon_1^2 + 2\epsilon_2^2)\mathcal{P} + 2(\epsilon_1 - \epsilon_2)^2 g^2\mathcal{P}^2\,.$$

The corresponding eigenvectors are obtained by using eq. (A.6) in eq. (A.3).

- An infinite tower of eigenvalues, $\mu_{\ell\pm} = |z_{\ell\pm}|/r$ which come in pairs such that $z_{\ell\pm} \to \pm c_\ell$ as $g \to 0$. This corresponds to a full tower of Kaluza Klein states whose masses are slightly perturbed away from the zeroeth-order result $|c_\ell|/r$ by the brane-bulk couplings.

A.2 Approximate symmetries II

We now present expressions for the eigenvalues and eigenvectors of the mass matrix in the case discussed in section 3.4. It is useful here to distinguish the following two subcases, as discussed in the main text:

(a) Unperturbed case: $g^+ = (0, g, g)^T$ and $g^- = (\epsilon, 0, 0)^T$

(b) Perturbed case: $g^+ = (-2g', g, g)^T$ and $g^- = (\epsilon, \epsilon', \epsilon')^T$

Case (a): unbroken lepton symmetry. The eigenvectors in this case resemble those found above:

- There is an eigenstate with zero eigenvalue:

$$\nu_0 = \frac{1}{\sqrt{2}} (0, 1, -1, 0, 0, 0, 0, \dots) .$$

- The rest of the eigenvectors have the form:

$$\nu_i = N_i (\alpha_i, 1, 1, x_{1i}, y_{1i}, x_{2i}, y_{2i}, x_{3i}, y_{3i}, \dots)$$

where

$$\begin{aligned} \alpha_i &= \frac{2\mathcal{S}'_i g\epsilon}{z_i (1 - \epsilon^2 \mathcal{P}'_i)}, \\ x_{\ell i} &= S'_{\ell i} \alpha_i \epsilon + z_i P'_{\ell i} 2g, \\ y_{\ell i} &= S'_{\ell i} 2g + z_i P'_{\ell i} \alpha_i \epsilon \end{aligned} \tag{A.7}$$

and the normalization factor N_i is defined as in eq. (A.5). As before, z_i is related to the mass eigenvalue μ_i by $\mu_i = |z_i|/r$, with z_i defined as the solutions to the eigenvalue equation:

$$z_i^2 \left(1 - \epsilon^2 \mathcal{P}'_i\right) \left(1 - 2g^2 \mathcal{P}'_i\right) - 2\, \mathcal{S}'^2_i g^2 \epsilon^2 = 0 \,, \tag{A.8}$$

with $\mathcal{P}'$ and $\mathcal{S}'$ defined as in eq. (A.2). The solutions to this equation come in two types:

- Two light eigenvalues given by

$$z_i = \pm 2 \sqrt{\frac{p_i}{u_i}}$$

with

$$\begin{aligned} u_i &= -2 - 2(\epsilon^2 + 2g^2)\mathcal{P} - 4\epsilon^2 g^2 \mathcal{P}^2 \\ p_i &= -(\epsilon g)^2 \mathcal{S}^2 \end{aligned} \tag{A.9}$$

where we have simplified the result as appropriate for those eigenvalues satisfying $z_i^2 \ll c_\ell^2$. The corresponding neutrino masses in this case do not depend on the label $\pm$ since they have the common mass $\mu_\pm = |z_\pm|/r$.

- Also in this case we have the tower of eigenstates whose eigenvalues (for small g) are perturbations to $z_{\ell\pm} \sim \pm c_\ell$.

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Case (b): broken lepton symmetry. Perturbing the above results with the addition of ϵ' and g' leads to a phenomenologically viable model. In this case we get the same structure of eigenvectors and eigenvalues, with the important difference that now the degeneracy between the lightest massive neutrinos is broken:

- One eigenstate remains massless, with eigenvector

$$\nu_0 = \frac{1}{\sqrt{2}}\,(0,1,-1,0,0,0,0,\ldots)\,.$$

- The rest of eigenvectors are given by:

$$\nu_i = N_i\,(\alpha_i,1,1,x_{1i},y_{1i},x_{2i},y_{2i},x_{3i},y_{3i},\ldots)\,, \tag{A.10}$$

where

$$\begin{aligned} \alpha_i &= \frac{2\mathcal{S}'_i\,(g\epsilon-2g'\epsilon')+2\mathcal{P}'_i z_i\,(-2gg'+\epsilon\epsilon')}{z_i\,(1-(4g'^2+\epsilon^2)\mathcal{P}'_i)+4g'\epsilon\mathcal{S}'_i}\,,\\ x_{\ell i} &= S'_{\ell i}\left(\alpha_i\epsilon+2\epsilon'\right)+z_i P'_{\ell i}\left(2g-2\alpha_i g'\right),\\ y_{\ell i} &= S'_{\ell i}\left(2g-2\alpha_i g'\right)+z_i P'_{\ell i}\left(\alpha_i\epsilon+2\epsilon'\right) \end{aligned} \tag{A.11}$$

with the normalization factor defined as in eq. (A.5). The eigenvalue equation in this case reads:

$$\begin{aligned} &\Big(z_i\left[1-(4g'^2+\epsilon^2)\mathcal{P}'_i\right]+4g'\epsilon\mathcal{S}'_i\Big)\Big(z_i\left[1-2(g^2+\epsilon'^2)\mathcal{P}'_i\right]-4g\epsilon'\mathcal{S}_i\Big)-\\ &\qquad -2\Big(\mathcal{S}'_i\left(g\epsilon-2g'\epsilon'\right)+\mathcal{P}'_i z_i\left(-2gg'+\epsilon\epsilon'\right)\Big)^2=0\,. \end{aligned} \tag{A.12}$$

The solutions to this equation have the same structure as before.

 - There are two light non-degenerated eigenvalues given by

$$z_i = \frac{v_i \pm \sqrt{v_i^2+4u_i p_i}}{u_i}\,, \tag{A.13}$$

 with

$$\begin{aligned} v_i &= -4(\epsilon g'-\epsilon' g)\mathcal{S}\\ u_i &= -2-2(\epsilon^2+2(\epsilon'^2+g^2+2g'^2))\mathcal{P}-4(\epsilon g+2\epsilon' g')^2\mathcal{P}^2\\ p_i &= -(\epsilon g+2\epsilon' g')^2\mathcal{S}^2\,, \end{aligned} \tag{A.14}$$

 where we simplify by assuming $z_i^2 \ll c_\ell^2$. As usual, the corresponding neutrino mass is $\mu_\pm = |z_\pm|/r$. The eigenvectors are obtained by using eq. (A.13) in eq. (A.10).

 - There is a tower of eigenstates organized in pairs, having eigenvalues which are perturbations of $z_{\ell\pm} \sim \pm c_\ell$.

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